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z-Scores and the Normal Curve

Plus a little probability thrown in for spice.

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I. z-scores and conversions

- What is a z-score?
  - A measure of an observation's distance from the mean.
  - The distance is measured in standard deviation units.
    - If a z-score is zero, it's on the mean.
    - If a z-score is positive, it's above the mean.
    - If a z-score is negative, it's below the mean.
    - If a z-score is 1, it's 1 SD above the mean.
    - If a z-score is –2, it's 2 SDs below the mean.

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Computing a z-score

$$z = \frac{X - \mu}{\sigma} \quad or \quad z = \frac{X - \overline{X}}{SD}$$

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Examples of computing z-scores

X	$\overline{X}$	$X-\overline{X}$	SD	$z=\frac{X-\overline{X}}{SD}$
5	3	2	2	1
6	3	3	2	1.5
5	10	-5	4	-1.25
6	3	3	4	.75
4	8	-4	2	-2

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Computing raw scores from z scores $X = z\sigma + \mu \quad or \quad X = zSD + \overline{X}$

$z=\frac{X-\overline{X}}{SD}$	SD	zSD	$\overline{X}$	X
1	2	2	3	5
-2	2	-4	2	-2
.5	4	2	10	12
-1	5	-5	10	5

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A-scores and T-scores

- z-scores have a mean of 0 and SD of 1
- T-scores have a mean of 50 and SD10
  - Gets rid of negative numbers.
  - Very commonly used in psychological scales, e.g., MMPI.
- A-scores have mean 500 and SD 100
  - Same deal. Used by SAT, GRE, etc.

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Moving between z and A
A=z*100+500; z=(A-500)/100

Z	Z*100	A	A	A-500	Z
0	0	500	500	0	0
1	100	600	600	100	1
-1	-100	400	550	50	.5
1.5	150	650	700	200	2
-.75	-75	425	675	175	1.75

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Moving between z and T
T=z*10+50; z = (T-50)/10

z	Z*10	T	T	T-50	z
0	0	50	50	0	0
1	10	60	60	10	1
-1	-10	40	55	5	.5
1.5	15	65	70	20	2
-.75	-7.5	42.5	67.5	17.5	1.75

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Moving between A and T

- A is 10 times bigger than T. Just slide that decimal point.
- If A = 600, then T=60.
- If T=40, then A=400.

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Review

- Interpret a z score of 1
- M = 10, SD = 2, X = 8. Z =?
- M = 8, SD = 1, z = 3. X =?
- What is the A (SAT) score for a z score of 1?

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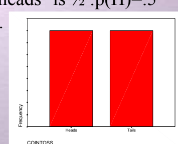
II. Probability

- Probability refers to the long run relative frequency of events.
- Example: The probability that a coin toss results in ‘heads’ is ½ :p(H)=.5 with a fair coin.



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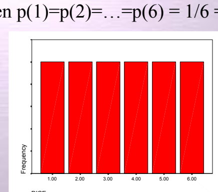
Probability Example 2

- Dice. A die has six sides. If it’s fair, then p(1)=p(2)=…=p(6) = 1/6 =.17.



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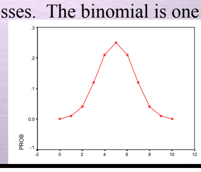
Binomial Distribution

- Mathematicians have figured formulas to estimate long run relative frequencies for simple events, like how many heads will appear for a given number of coin tosses. The binomial is one such.



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Binomial Distribution

- Relative frequency or probability is associated with the shape of the curve. The height of the curve shows the relative frequency. Binomial is discrete.
- Notice that the binomial approximates the bell curve, especially over lots of trials (the distribution of 'heads' for 20 trials would look more normal than for 10 trials).

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Normal Curve

- The normal curve is continuous.
- The formula is: $Y = \frac{N}{\sigma\sqrt{2\pi}} e^{-(X-\mu)^2/2\sigma^2}$
- This formula is not intuitively obvious.
- The important thing to note is that there are only 2 parameters that control the shape of the curve: σ and μ. These are the population SD and mean, respectively.

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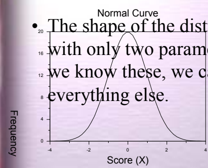
Normal Curve

- The shape of the distribution changes with only two parameters, σ and μ, so if we know these, we can determine everything else.



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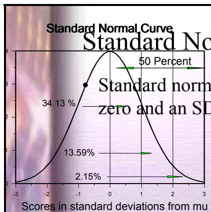
Standard Normal Curve

- Standard normal curve has a mean of zero and an SD of 1.



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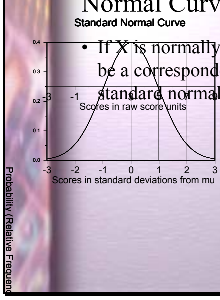
Normal Curve and the z-score

- If X is normally distributed, there will be a correspondence between the standard normal curve and the z-score.



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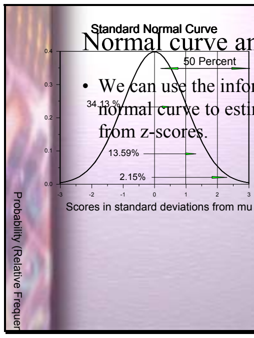
Normal curve and z-scores



- We can use the information from the normal curve to estimate percentages from z-scores.

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Test your mastery of z

- If a raw score is 8, the mean is 10 and the standard deviation is 4, what is the z-score?
- 1: -1.0
- 2: -0.5
- 3: 0.5
- 4: 2.0

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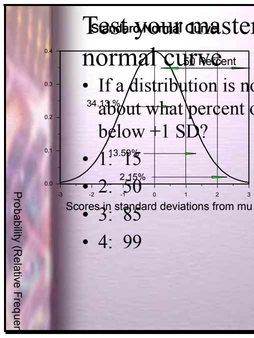
Test your mastery of z and the normal curve



- If a distribution is normally distributed, about what percent of the scores fall below +1 SD?
- 1: 15
- 2: 50
- 3: 85
- 4: 99

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Tabled values of the normal to estimate percentages

Z	Between mean and z	Beyond z		Z	Between mean and z	Beyond z
0.00	0.0	50.00		0.90	31.5	18.41
0.10	3.98	46.02		1.00	34.13	15.87
0.20	7.93	42.07		1.10	36.43	13.57
0.30	11.79	38.21		1.20	38.49	11.51
0.40	15.54	34.46		1.30	40.32	09.68
0.50	19.15	30.85		1.40	41.92	08.08
0.60	22.57	27.43		1.50	43.32	06.68
0.70	25.80	24.20		1.60	44.52	05.48
0.80	28.81	21.19		1.70	45.54	04.46

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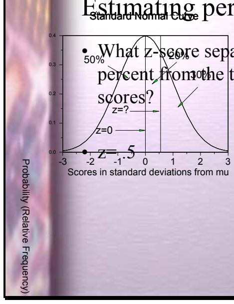
Estimating percentages



- What z-score separates the bottom 70 percent from the top 30 percent of scores?
- z=.5

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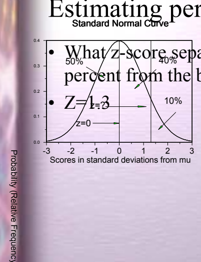
Estimating percentages



- What z-score separates the top 10 percent from the bottom 90 percent?
- Z=1.3

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Percentile Ranks

- A percentile rank is the percentage of cases up to and including the one in which we are interested. From the bottom up to the current score.
- Q: What is the percentile rank of an SAT score of 600?

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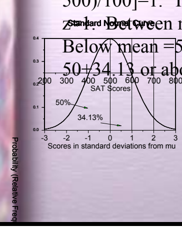
Percentile Rank

- A: First we find the z score [(600-500)/100]=1. Then we find the area for z=1. Between mean and z = 34.13. Below mean =50, so total below is 50+34.13 or about 84 percent.



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Estimating percentages

- Suppose our basketball coach wants to estimate how many entering freshmen will be over 6’6” (78 inches) tall. Suppose the mean height of entering freshmen is 68 inches and the SD of height is 6.67 inches and there will be 1,000 entering freshmen. How many are expected to be bigger than 78 inches?

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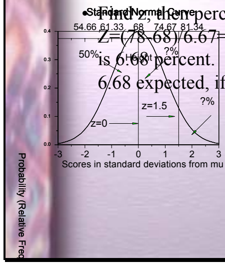
Estimating percentages

- Find z, then percent, then the number. Z=(78-68)/6.67=1.499=1.5. Beyond z is 6.68 percent. If 100 people, would be 6.68 expected, if 1000, 66.8 or 67 folks.



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Review

- What z score separates the top 20 percent from the bottom 80 percent?
- What is a percentile rank?
- Suppose you want to estimate the percentage of women taller than the height of the average man. Say M_{male} = 69 in. M_{female} = 66 in. SD_{female}= 2 in. Pct?

  Z = (69-66)/2 = 3/2 = 1.5

  Beyond z = 1.5 is 6.68 pct.